# Separating periods of quartic surfaces

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# Periods to algebraic cycles

Periods = { Integrals of algebraic functions over algebraic domains}

$$\pi = 2\int_{-1}^{1} \sqrt{1-x^2}\,\mathrm{d}x$$

$a, b \in$ Periods, $a = b \iff \exists$ an algebraic cycle in a variety over $\overline{\mathbb{Q}}$

$X = Z(f) \subset \mathbb{P}^3$, $f \in \mathbb{Z}[x,y,z,w]_4$, $\omega_f \in \mathrm{H}^{2,0}(X/\mathbb{Q})$

$$\text{Periods of } X = \left\{ \int_\gamma \omega_f \;\middle|\; \gamma \in \mathrm{H}_2(X,\mathbb{Z}) \right\}.$$

Lefschetz (1,1)-theorem: $\int_\gamma \omega_f = 0 \iff \gamma = [C_1] - [C_2]$

## An analogy with algebraic numbers

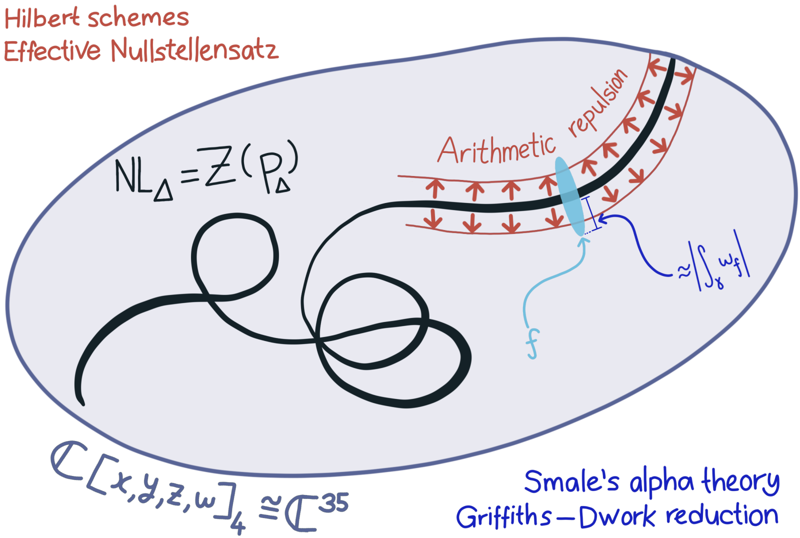
| | | |
|---|---|---|
| $\alpha \in \overline{\mathbb{Q}}$: | (min poly., approx.) | degree, height |
| $\int_\gamma \omega_f$: | $(f, \gamma)$ | $\Delta_\gamma$, $\lvert f \rvert$. |

There is an effective constant $\varepsilon(\Delta_\gamma, f)$ such that:

$$\int_\gamma \omega_f = 0 \quad \text{or} \quad \left| \int_\gamma \omega_f \right| > \varepsilon(\Delta_\gamma, f).$$

$$\varepsilon(\Delta, f) = \frac{\left( d_\Delta! |p_\Delta| \left(1 + |f|\right)^{d_\Delta} \right)^{-1}}{4 \| \mathrm{d}\mathcal{P}_f^{-1} \| \left( 1 + 6 \frac{\|A\|}{\mathrm{vol}(X_f)} \right)}$$

$\sum_{n \geq 0} \left(2 \uparrow\uparrow (3^n)\right)^{-1}$ is not a ratio of periods of a quartic surface/$\mathbb{Q}$.


Hilbert schemes
Effective Nullstellensatz
$NL_\Delta = Z(P_\Delta)$
Arithmetic repulsion
$f$
$\approx \left| \int_\gamma \omega_f \right|$
$\mathbb{C}[x,y,z,w]_4 \cong \mathbb{C}^{35}$
Smale's alpha theory
Griffiths–Dwork reduction